Games and Information

A review of Rasmusen (2006)

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Chapter 1

Introduction

1.1 The book itself

This review focuses on the book *"Games and Information: An introduction to Game Theory"* by Eric Rasmusen [1]. It was first published in 1989 and has seen several updates. For this review the fourth edition is discussed, published in 2007 (ISBN: 1405136669). While not all Internet retailers have the book in stock, it can usually be ordered from most common web stores where its price-tag of about 40 euros make it comparatively cheap for a textbook.

"Games and Information" can be considered a typical introductory text. It discusses topics and areas of interest in game theory and economics, divided in three parts: game theory, asymmetric information, and applications. For this review, Part II on asymmetric information will be mostly ignored, to focus on chapters 1 through 6 (game theory) as well as chapters 12 and 13 (applications).

The content of the book is mostly comprised of concepts and theories relating to the fields of game theory (including applications in economics and law, amongst others), augmented by examples and more in-depth discussions of some concepts. An observant reader will note that many of these examples originate from earlier publications, simplified sometimes for clarity.

While a university level education is assumed, surprisingly little prior knowledge is required to understand the book. It is accessible to students with a minimum of experience in game theory, although some concepts may be used in an unfamiliar setting and, as such, may require the use of additional sources to understand their meaning.

1.2 This review

The bulk of this review consists of two parts: a report of the book's contents, followed by an evaluation discussing the former. It is intended to give a fair overview of the book's applicability as textbook for a course in game theory to comparatively novice users, as well as to give an overview of its contents without being an actual summary. In places references are given to articles and books the content is based upon, enabling someone familiar with the field to estimate the general extent of the material.

The intended audience for this review can therefore be summarized as everyone considering to purchase the book: it should allow the reader to determine whether the topics match his or her interests and, in addition, provide an informed opinion on its intended applicability as textbook.

Chapter 2

The book, chapter by chapter

2.1 Game Theory

2.1.1 The Rules of the Game

In Chapter 1 of the book, Rasmusen starts by giving common definitions and concepts used in game theory. The essential elements of a game are introduced, including *players, actions, payoffs,* and *information*, as well as how these combine to form *strategies, equilibria,* and an *outcome*.

Rasmusen claims that in modelling a real-world situation, finding the payoffs is often the hardest part. He elaborates on this, together with the randomness from a move by *nature*, through a payoff matrix for the *Dry Cleaners Game*. In this game a new company decides to either *enter* a new market or *stay out*, while an existing company must choose between *colluding* with the new player or *fighting* him.

A *decision tree* of this game is used to introduce the *order of play* aspect, depicting the choices for a single player. In contrast, a *game tree* is used to illustrate the entire game.

From the game tree a player's *strategy* can be derived: a rule to tell the player which action to choose. A *strategy profile* is a list containing one strategy for each player in the game.

The best strategies combine into an *equilibrium*, whose outcome is an *equilibrium outcome*. A *dominant-strategy equilibrium* is an equilibrium containing each player's *dominant strategy*, that is, the strictly best response for each player to any strategies the other players might pick.

In a *cooperative game*, players can make binding commitments which can change these strategies. Alternatively, *side-payments* can change the payoffs with the same result.

An *iterated-dominance equilibrium* can sometimes be found by eliminating weakly dominated strategies. A *Nash equilibrium* is a strategy profile from which no player wishes to deviate, given that the other players do not deviate. In *coordination games* multiple Nash equilibria exist, requiring players to coordinate on a single equilibrium.

Comments: Little controversy exists in this first chapter, as Rasmusen mostly introduces common concepts in game theory without going into much detail. Not all definitions seem logically placed however: the definition of *pseudo-players* for example seems random and the concept is not mentioned again, leaving the reader to wonder why it was introduced in the first place.

In contrast, relatively much attention is given to *game trees*, whereas *strategies* have to make do with a single paragraph. Similarly, a great deal of attention is paid to coordination games despite their recurrence later on in the book. A more consistent approach would see the bulk of that elaboration moved to Chapter 3 for example, where coordination games are discussed in detail.

This non-homogeneous distribution of detail to differing aspects of game theory gives Chapter 1 an unpolished appearance. Overall though, the chapter serves well to make clear the elemental aspects of games, despite at times being as erratic as a tourist visiting madurodam for the first time.

2.1.2 Information

In Chapter 2, Rasmusen focuses on information. He introduces the *strategic form*, which consists of all possible strategy profiles and payoff functions, and the *outcome matrix*: all possible action profiles and their outcome functions.

The *extensive form* is introduced as variation on the *game tree*. It is refined to include *information sets* within dotted lines. A player's *information partition* is now defined as a collection of his information sets.

Different types of information are explained: *common knowledge* is known to all players, and all players know that all players know that information, and all players know that all players know that all players know that information, et cetera ad infinitum. For *perfect information* every information set is a *singleton*. A game of *certainty* prohibits moves by Nature after any player moves. In a *symmetric information* game, players effectively know the same, unlike a game of *incomplete information*, defined by Rasmusen as a game where Nature moves first and is unobserved by at least one of the players.

A *Harsanyi transformation* transforms a game of incomplete information into a game of complete but imperfect information. Through *Bayes' Rule*, players can then update their belief of being in a certain node.

Comments: When reading Chapter 2, one gets the feeling that visualizing might not actually be Rasmusen's strong suit. It starts with Table 2.2 which requires a notepad to work out, and does not improve when he attempts to explain different aspects of a game tree without actually providing a game tree for reference. Ironically he does put focus on improper visualizations where time lines are concerned.

As the chapter progresses, the style becomes clearer, however, and Section 3 is particularly clear and well written. To offset this, Section 4 casts doubt on former definitions, though without causing more confusion than the field itself contains on those definitions; the reader is left with the distinct impression that

Rasmusen tries to apologize for game theory being somewhat disorganized at times. Section 5 introduces the grey box-style used throughout the book to denote examples, and a surprisingly well worked out example serves as a neat finish to a coherent (albeit impromptu) introduction to information.

2.1.3 Mixed and continuous strategies

In Chapter 3, Rasmusen removes one simplification in his examples so far by allowing a continuum of moves. This also introduces *mixed strategies* which map a player's information sets to probability distributions.

To determine the payoffs from a mixed strategy, Rasmusen proposes the *payoff-equating* method which states that in a mixed strategy, each pure strategy must yield the same payoff.

If players use the same randomizing device for their mixed strategies this is said to yield *correlated strategies*. If this device is modeled, the resulting uncertainty is called *extrinsic uncertainty*. Another proposed method for coordination is *cheap talk* [2]: a form of non-committing communication which enables players to coordinate on a specific equilibrium.

Rasmusen claims that the three most prominent ways to model continuous strategies in a *duopoly* (a market with two players) are Cournot [3], Stacker [4] and Bertrand [5]. Finding their equilibria requires a *best-response function*.

Comments: The length of the description above might give the observant reader of this review a clue as to the contents of Chapter 3: surprisingly little. There are two concepts Rasmusen wants the reader to understand after working through the chapter, and he is making absolutely sure they are understood by creating, in essence, one huge example. Perhaps this is for the best: mixed and continuous strategies are complex, even more so than the examples make you believe, and giving them a great deal of attention is prudent. Focusing so long

on a single concept also makes Chapter 3 arguably the least interesting chapter so far.

2.1.4 Dynamic games with symmetric information

Chapter 4 introduces *subgame perfectness*. A *subgame perfect Nash equilibrium* is a Nash equilibrium for the entire game as well as for every *subgame*. Rasmusen describes a subgame as follows:

> A subgame is a game consisting of a node which is a singleton in every player's information partition, that node's successors, and the payoffs at the associated end nodes.

It should be noted that this definition can include the entire game as subgame, a property sometimes required when no other node is a singleton in every player's information partition, as Rasmusen discusses in more detail in Chapter 6.

Subgame perfectness implies that a player should reoptimize his decisions at each point in the game, a concept often called *sequential rationality*. This is more robust when there is a small probability of the opponent making a mistake, called a *tremble*.

Other variations on the Nash equilibrium exist as well: a *coalition-proof Nash equilibrium* for example is a Nash strategy profile from which no coalition of players would deviate. It was introduced by Bernheim [6] to handle situations where no *Strong Nash Equilibrium* exists.

The parameters of a dynamic game can be optimized to adjust the outcome of the game. An example of this is given by Janssen [7], where a firm sets an offered wage such that the dynamic equilibrium converges to a point where only high potential workers apply for a job. He demonstrates that in some situations, a subgame perfect Nash equilibrium exists, which can be manipulated through varying the payoffs at each end node, to induce only specific players to signal.

This then serves as a form of selection in a job market.

It should be noted, however, that most models here described ignore the human factor. Harlé and Sanfey demonstrate that emotion can play an important role in decision making [8] and whatever equilibrium concept is used, the equilibrium outcome is not always obtained in real world scenarios.

Comments: What can you say about a mere twenty pages introducing a simple topic, subgames, in their most simplistic form: those with symmetric information? Having co-taught this chapter for three hours I can safely claim: quite a lot. Rasmusen luckily does not appear to share in this conviction and finishes off with a few examples and the feeling that Chapter 4 is little more than an introduction to the real work which is to come in Part II. Sadly, in doing so he neglects to pay attention to some of the more interesting aspects of symmetrical subgames. Ethics and morality can come into play here, as illustrated for example by the *Doomsday Device*[*] and the game of *Entry Deterrence*.

2.1.5 Reputation and repeated games with symmetric information

Chapter 5 focuses on a special class of dynamic games: repeated games. Rasmusen starts to explain the *Chainstore paradox*, where a game is repeated a finite number of times, effectively reducing it to a *one-shot game* played just once.

The *Folk Theorem* claims that for infinitely repeated games, any observed strategy is the unique outcome of a subgame perfect equilibrium given a number of conditions. This depends in part on the *minimax payoff*, the maximum payoff a player can achieve if all other players attempt to minimize his outcome, based on the *Minimax theorem* [9].

[*]A *Doomsday Device*[†]is a weapon that can anihilate all life on a planet, triggered to activate automatically whenever its builders are under attack.

[†]As explained in the incredible movie *Dr. Strangelove*, 1964, by Stanley Kubrick.

Rasmusen gives examples of different strategies in repeated games: the *Markov strategy*, *Evolutionarily stable strategy* and the *bourgeois strategy* are discussed.

Comments: Repetition in games is a complex topic, a problem Rasmusen attempts to overcome by giving it a great deal of attention: of the chapters in Part I, only Chapter 3 is longer. However, perhaps even this amount of attention is not sufficient to clarify the concepts introduced here; the *Folk Theorem* concept, for example, is followed by two pages explaining its conditions, but the reader is left wondering what the theorem can actually be used for. The chapter then continues with some related concepts (like *minimax*) before diving into examples, without properly explaining all the details of repetition in games.

Particularly in the last of these examples one cannot help but get the impression that Rasmusen's reluctance to *talk math* came at a price: not all of the theory is conveyed as clearly as it could be. The advantages of keeping the section somewhat abstract are clear however: the example illustrates perfectly the result of (infinite) repetition in mixed strategies. As such, one wonders why the section is deemed "less important" by the author, as it greatly augments understanding the theory discussed.

2.1.6 Dynamic games with incomplete information

In Chapter 6, asymmetric and incomplete information is considered. Rasmusen claims this is especially important in dynamic games where early moves may convey private information.

When a lack of singletons precludes a subgame perfect Nash equilibrium, a *trembling-hand perfect equilibrium* may be applicable. An alternative approach is the *perfect Bayesian equilibrium*. In order to form beliefs, *passive conjectures* can retain the prior.

Even when information is not complete, certain strategies relying on others believing uncertain information can still be successful. This is explained

through reputation in what Rasmusen calls the *the Gang of Four model*, named after Kreps, Milgrom, Roberts & Wilson. In another famous example, Axelrod's arguments for Tit-for-Tat in the prinsoners' dilemma are explained: *niceness*, *provocability* and *forgiving* make it particularly successful when a player's strategy is uncertain. Putting theory to practice, Rapoport used Tit-for-Tat in the *Axelrod Tournament*, winning twice.

Comments: The general idea behind Chapter 6 is perhaps just this: even when information is incomplete, it can still be used or inferred. Strategies should take this into account in dynamic games with incomplete information. A bold claim, but made sufficiently plausible through the use of multiple examples. "Incomplete information" is also an apt title for the chapter: the reader is left with the distinct impression that in games with incomplete information, there might just not be a clear and well defined equilibrium. Perhaps this is so; or just perhaps the text could emphasize more on what can be done to obtain certainty in these situations, if at all possible.

2.3 Applications

2.3.12 Bargaining

The first application which Rasmusen discusses in Chapter 12 is bargaining. An already mentioned example is the *Ultimatum Game* where a pie is split; in real world experiments, emotions make the outcome seldom identical to the simple symmetrical pure-strategy equilibrium. Going backwards from a likely outcome is exemplified by the *Nash bargaining solution*, conforming to four axioms: *invariance*, *efficiency*, *independence of irrelevant alternatives* and *anonymity*.

In the *double auction mechanism* example, two Nash equilibria are discussed: the *one-price equilibirum* and the *linear equilibrium*. The latter is compared with two mechanisms which induce truth-telling: *the Chatterjee-Samuelson mecha-*

nism and a *direct incentive-compatible mechanism* where truth-telling does not penalize players.

Comments: Chapter 12 comes across as somewhat of a mix: a great many examples illustrate various trading mechanisms and their respective equilibria. Overall these are well written and quite clear, though given less attention than often required for more in-depth discussion of each mechanism. There can be little critique on the chapter as a whole, except possibly that when all is said and done, its actual theoretical content is minimal. This is understandable (since Part III is, after all, called applications), yet more obvious here than in, for example, Chapter 13.

2.3.13 Auctions

Chapter 13 concerns an alternate method for purveying goods: auctions. Rasmusen claims that auctions are well suited for modelling since they are stylized markets with well-defined rules. These rules differ however between different types of auctions, requiring intricate classification.

The value of the goods to be auctioned is private to each bidder in a *private-value auction*, as opposed to common knowledge in a *common-value auction*. A further distinction is made between the case where a bidder's value is not related to other valuations (*independent private-value auction*) and the case where values are related (*affiliated private-value auction*). Rasmusen claims that in practice a combination of private and common valuations is more likely, noting that the term *interdependent value* has been used to denote this scenario.

Five common auction types are discussed in detail: *ascending*, *first price*, *second price*, *descending* and *all pay*. Analyzing their equilibria yields interesting results, noting for instance that the descending auction is *strategically equivalent* to the first-price auction. A possibly more interesting result, however, is the *Revenue Equivalence Theorem* [10] introduced next, which roughly states that

under certain conditions, the seller's expected revenue is the same for all auction types.

To influence this expected revenue, a seller can use a *reserve price* which sets a minimum value. Through the *marginal revenue approach* it can be seen that the seller's payoff increases, regardless of the number of bidders.

Comments: What Chapter 12 is for bargaining, Chapter 13 aspires to be for auctions: an overview and an example, with just enough in-depth discussion to appeal to the interest of economy students. It also fails to overcome the problem in Chapter 12: either go into enough detail to be applicable or stick to illustrating the theory. It is tempting to give the benefit of the doubt here: the chapter most certainly increased my understanding of auctions and demonstrated that they are much more complex than I had hitherto assumed.

Having said that, Rasmusen does not convince that the theories actually work and the proposed equilibria also exist in real world scenarios. Since it is unlikely that the field is totally bereft of any link to the real world (quite unlike astrophysicists for example, who revel in probing the theoretical boundaries of existence) the reader is left with the impression that something is missing. Rasmusen makes up for this and more in his chapter-notes, however, which are particularly useful for this chapter.

Chapter 3

Evaluation

3.1 Structure

When evaluating *Games and Information* it is impossible to ignore its main purpose and intended audience. The book is obviously written for students, and serves as an introduction to game theory for academics. It should therefore come as little surprise that its content and structure are uncontroversial: Rasmusen maintains a clear writing style with surprisingly little variance in difficulty level.

By maintaining a consistent style augmented with a great many examples, Rasmusen manages to keep all topics he discusses accessible for a semi-novice user. Having said that, where applications are concerned, in Part III, the use of mathematics is not shunned*, and this may dissuade a less mathematically inclined reader to work through all the examples. The case is not helped by the greater length of the chapters in this part: where Chapter 4 for example lasts a mere 20 pages, more than double that is taken up by Chapter 13 at 48 pages. It is up to the reader to divide this into manageable chunks, arguably not an easy task for every novice game theorist.

Throughout the book, Rasmusen maintains a similar structure: introduce a novel topic (possibly mediated by related terms in previous chapters), work out

*Or at least, less so than in Part I.

one or more examples, then discuss more complex variations thereof. As one advances in the book, previous topics are considered known to the reader and a chronological progression is advisable for new users, reducing the ability to teach just specific topics, as some textbooks allow. Each chapter is concluded by a series of assignments, as well as notes by the author. The use of chapter notes is of course not unique[†], but decidedly useful and noteworthy in this case.

3.2 Content

The topics that are discussed give the impression to have been selected in order to promote internal coherence, rather than give as broad a view of the area of interest as possible. This is particularly obvious in Part III, applications, which focuses entirely on economic applications of game theory (more specifically: trading). While the definition of "games" in the field of game theory is obviously stretched somewhat from what the general public assumes it to mean, one nonetheless gets the feeling that, for example, war-games or political games deserved some attention. While some examples are given (eg., the battle of the Bismarck sea), they are rare and far apart.

Another aspect which is largely ignored is real world applicability of the given theories. In the discussion of the *Ultimatum Game*[‡] for example, Rasmusen notes the possibility of ill-will affecting the equilibrium but leaves it at that. Actual research, however, shows that offers of up to 30% of the to be shared amount are typically rejected [8], a significant deviation from what one might expect from merely reading the book.

Likewise, most of the examples have been simplified considerably. While this makes them much clearer and arguably helps to bring the point across, the impression is given that finding an equilibrium is as easy as looking for a simple pay-off equation. In real world scenarios, however, even a simple equilibrium

[†]See for example the book *Vector Calculus* [11], or one of my favourites, *Fundamentals of Computational Neuroscience* [12].

[‡]See Chapter 12, Section 1.

can require several pages of mathematical induction to obtain [7]. Rasmusen's tendency to simplify the mathematical background of much of the theory effectively means that anyone looking to apply the material will find that additional textbooks are required to learn the actual implementations of the theory. While this is quite understandable in an introductory text, it is nonetheless regrettable.

3.3 Style

There can be little doubt that Rasmusen attempts to maintain a clear narrative tone for much of the book's contents. While somewhat brave to attempt such difficult a task, there is no denying that including tables, formulas and the occasional example box breaks the flow of text somewhat, leaving *Games and Information* just a bit short of actually being a well-written narrative. The effect can be likened to listening to a powerpoint presentation by someone not actually used to giving presentations: a good story, broken up just when you get drawn in by a block of graphs and tables as subtle as a flash-light in a dark cavern. Considering that a textbook is actually supposed to teach you something, however, Rasmusen can easily be excused as having done an excellent job in a difficult situation.

Gray blocks of examples punctuate an otherwise quite interesting story. Their clearly recognizable structure is a testament to much of the book content's consistency: examples, questions, graphs, tables, all have a distinct style maintained throughout the text, greatly improving the overall readability. Compared to such horrors of textbook literature as *Radiative Processes in Astrophysics* [13] it is truly well written, and manages not unlike Moore's excellent introduction to the theory of special relativity[§] to introduce a difficult concept in such a clear and well-written style that a determined reader will go through the material easily, feeling enlightened at the end, simply wondering: "is that really all there is to it?".

[§]I am, of course, referring to *A Traveler's Guide to Spacetime* [14].

Chapter 4

Conclusion

It is difficult for a novice reader to give a final judgment on a text without knowing precisely how it compares to other books in the field (which, if I had read them, would indubitably disqualify me as a novice reader); however, in the spirit of millions of Internet forum users around the globe I will now attempt to do just that.

Games and Information is, for all intents and purposes, a classical textbook. It contains plenty of examples, theories, counter-theories and concepts to familiarize a student with the field of game theory. Like all such texts it is not complete, nor does it contain enough depth to be practically useful for any real implementations. This is an inherent problem with any introductory text, and Rasmusen manages to find an optimum balance few others achieve. The writing is clear and easy to follow, yet the flow of the narrative is continuously broken by all those extra pieces of content a textbook requires.

For all that, I highly doubt there are significantly better textbooks on game-theory in existence, if any better texts exist at all*. *Games and Information* manages to obtain a perfect equilibrium in its contents, going to just the right

*The observant game theorist will now of course note that, if there are better texts available, the same instructors grading this review are most likely either unaware of this, or disagree that another text is better. Would my pay-off then decrease by pointing out their error? What does that imply about the reviews made by any student who paid attention in class? But I digress...

depth where possible and selecting exactly those topics most likely to be of interest to a novice game theorist. A course coordinator cannot go far wrong selecting this book for a course in game theory, and his or her students will be the better for it.

Was signed,

Matthijs Dorst;

Slightly less novice reader

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